

1 次の行列式を計算せよ.

$$(1) \begin{vmatrix} 12 & 9 \\ 13 & 8 \end{vmatrix} \quad (2) \begin{vmatrix} 0 & 0 & 10 \\ 0 & -7 & 11 \\ 5 & 8 & -12 \end{vmatrix} \quad (3) \begin{vmatrix} 1 & -3 & 7 \\ 0 & 5 & 9 \\ 2 & -11 & 13 \end{vmatrix} \quad (4) \begin{vmatrix} 1 & 0 & -2 & -1 \\ 1 & 1 & -2 & 3 \\ 0 & 2 & -1 & 0 \\ -1 & 2 & -1 & 2 \end{vmatrix}$$

$$(5) \begin{vmatrix} 2 & 0 & 2 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 0 & 2 \end{vmatrix} \quad (6) \begin{vmatrix} 0 & 1 & -2 & 1 \\ -1 & 0 & -1 & 0 \\ -2 & 1 & 0 & 1 \\ 1 & -1 & 2 & 0 \end{vmatrix} \quad (7) \begin{vmatrix} 2 & 3 & -3 & 5 \\ -3 & 0 & 1 & 0 \\ 3 & -1 & -3 & 3 \\ 4 & 5 & -2 & 5 \end{vmatrix}$$

2 行列 $A = \begin{pmatrix} -1 & -2 & 2 \\ 0 & 1 & -3 \\ 2 & -2 & 3 \end{pmatrix}$ とする.

(1) A の (i, j) 余因子 Δ_{ij} ($1 \leq i, j \leq 3$) を (i, j) 成分とする 3 次行列 $B = (\Delta_{ij})$ を求めよ.

(2) A の行列式 $|A|$ の値を求めよ.

(3) A の逆行列 A^{-1} を求めよ.

3 次の行列式を因数分解せよ.

$$(1) \begin{vmatrix} a & a & b \\ 2a & a+b & 2b \\ 2a+b & a+2b & 3a \end{vmatrix} \quad (2) \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} \quad (3) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \quad (4) \begin{vmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{vmatrix}$$

第12回演習問題の解答：

1 次の行列式を計算せよ。

$$(1) \begin{vmatrix} 12 & 9 \\ 13 & 8 \end{vmatrix} \xrightarrow{\textcircled{2}-\textcircled{1}} \begin{vmatrix} 12 & 9 \\ 1 & -1 \end{vmatrix} = 3 \begin{vmatrix} 4 & 3 \\ 1 & -1 \end{vmatrix} = 3(4 \times (-1) - 3 \times 1) = -21$$

$$(2) \begin{vmatrix} 0 & 0 & 10 \\ 0 & -7 & 11 \\ 5 & 8 & -12 \end{vmatrix} \xrightarrow{\textcircled{1} \leftrightarrow \textcircled{3}} \begin{vmatrix} 5 & 8 & -12 \\ 0 & -7 & 11 \\ 0 & 0 & 10 \end{vmatrix} = -5 \times (-7) \times 10 = 350$$

$$(3) \begin{vmatrix} 1 & -3 & 7 \\ 0 & 5 & 9 \\ 2 & -11 & 13 \end{vmatrix} \xrightarrow{\textcircled{3}-2 \times \textcircled{1}} \begin{vmatrix} 1 & -3 & 7 \\ 0 & 5 & 9 \\ 0 & -5 & -1 \end{vmatrix} \xrightarrow{\textcircled{3}+\textcircled{2}} \begin{vmatrix} 1 & -3 & 7 \\ 0 & 5 & 9 \\ 0 & 0 & 8 \end{vmatrix} = 1 \times 5 \times 8 = 40$$

$$(4) \begin{vmatrix} 1 & 0 & -2 & -1 \\ 1 & 1 & -2 & 3 \\ 0 & 2 & -1 & 0 \\ -1 & 2 & -1 & 2 \end{vmatrix} \xrightarrow{\begin{matrix} \textcircled{2}-\textcircled{1} \\ \textcircled{4}+\textcircled{1} \end{matrix}} \begin{vmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 0 & 4 \\ 0 & 2 & -1 & 0 \\ 0 & 2 & -3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 4 \\ 2 & -1 & 0 \\ 2 & -3 & 1 \end{vmatrix} \xrightarrow{\begin{matrix} \textcircled{2}-2 \times \textcircled{1} \\ \textcircled{3}-2 \times \textcircled{1} \end{matrix}} \begin{vmatrix} 1 & 0 & 4 \\ 0 & -1 & -8 \\ 0 & -3 & -7 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -8 \\ -3 & -7 \end{vmatrix} = \begin{vmatrix} 1 & 8 \\ 3 & 7 \end{vmatrix} = 1 \times 7 - 8 \times 3 = 7 - 24 = -17$$

$$(5) \begin{vmatrix} 2 & 0 & 2 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 0 & 2 \end{vmatrix} \xrightarrow{\textcircled{2}+\textcircled{1}} 2 \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 0 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 2 \end{vmatrix}$$

$$= 2(-1^2 \times (-1) - 1 \times 2^2) = 2 \times (-3) = -6$$

$$(6) \begin{vmatrix} 0 & 1 & -2 & 1 \\ -1 & 0 & -1 & 0 \\ -2 & 1 & 0 & 1 \\ 1 & -1 & 2 & 0 \end{vmatrix} \xrightarrow{\textcircled{1} \leftrightarrow \textcircled{2}} \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 \\ -2 & 1 & 0 & 1 \\ 1 & -1 & 2 & 0 \end{vmatrix} \xrightarrow{\begin{matrix} \textcircled{3}-2 \times \textcircled{1} \\ \textcircled{4}+\textcircled{1} \end{matrix}} \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & 1 & 0 \end{vmatrix}$$

$$= -(-1) \begin{vmatrix} 1 & -2 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix} \xrightarrow{\begin{matrix} \textcircled{2}-\textcircled{1} \\ \textcircled{3}+\textcircled{1} \end{matrix}} \begin{vmatrix} 1 & -2 & 1 \\ 0 & -4 & 0 \\ 0 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ -1 & 1 \end{vmatrix} = 4 \times 1 = 4$$

$$(7) \begin{vmatrix} 2 & 3 & -3 & 5 \\ -3 & 0 & 1 & 0 \\ 3 & -1 & -3 & 3 \\ 4 & 5 & -2 & 5 \end{vmatrix} \xrightarrow{\textcircled{1}+3 \times \textcircled{2}} \begin{vmatrix} -7 & 3 & -3 & 5 \\ 0 & 0 & 1 & 0 \\ -6 & -1 & -3 & 3 \\ -2 & 5 & -2 & 5 \end{vmatrix} \xrightarrow{\textcircled{2} \text{で展開}} 1 \times \left(\begin{vmatrix} -7 & 3 & 5 \\ -6 & -1 & 3 \\ -2 & 5 & 5 \end{vmatrix} \right)$$

$$\xrightarrow{\begin{matrix} \textcircled{1}+3 \times \textcircled{2} \\ \textcircled{3}+5 \times \textcircled{2} \end{matrix}} \begin{vmatrix} -25 & 0 & 14 \\ -6 & -1 & 3 \\ -32 & 0 & 20 \end{vmatrix} \xrightarrow{\textcircled{2} \text{で展開}} -(-1) \begin{vmatrix} -25 & 14 \\ -32 & 20 \end{vmatrix} \xrightarrow{\textcircled{2}-\textcircled{1}} \begin{vmatrix} -25 & 14 \\ -7 & 6 \end{vmatrix}$$

$$\xrightarrow{\textcircled{1}+\textcircled{2}} \begin{vmatrix} -11 & 14 \\ -1 & 6 \end{vmatrix} = (-11) \times 6 - 14 \times (-1) = -52$$

$$\textcircled{2} (1) B = \begin{pmatrix} + \begin{vmatrix} 1 & -3 \\ -2 & 3 \end{vmatrix} & - \begin{vmatrix} 0 & -3 \\ 2 & 3 \end{vmatrix} & + \begin{vmatrix} 0 & 1 \\ 2 & -2 \end{vmatrix} \\ - \begin{vmatrix} -2 & 2 \\ -2 & 3 \end{vmatrix} & + \begin{vmatrix} -1 & 2 \\ 2 & 3 \end{vmatrix} & - \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} \\ + \begin{vmatrix} -2 & 2 \\ 1 & -3 \end{vmatrix} & - \begin{vmatrix} -1 & 2 \\ 0 & -3 \end{vmatrix} & + \begin{vmatrix} -1 & -2 \\ 0 & 1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -3 & -6 & -2 \\ 2 & -7 & -6 \\ 4 & -3 & -1 \end{pmatrix}$$

(2) $|A| = 11$

(3) $A^{-1} = \frac{1}{11} \begin{pmatrix} -3 & 2 & 4 \\ -6 & -7 & -3 \\ -2 & -6 & -1 \end{pmatrix}$

3 (1) $-(a-b)^2(3a+b)$ (2) $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ (3) $(x-y)(y-z)(z-x)$
 (4) $-a(a-b)(b-c)(c-d)$