

1 行列  $A = \begin{pmatrix} 2 & 3 & 0 \\ -1 & 4 & 1 \\ -3 & 0 & 2 \end{pmatrix}$  の  $(i, j)$  余因子  $\Delta_{ij}$  ( $1 \leq i, j \leq 3$ ) を全て求めよ. (9 点)

$$\begin{aligned} \Delta_{11} &= + \begin{vmatrix} 4 & 1 \\ 0 & 2 \end{vmatrix} = 8, & \Delta_{12} &= - \begin{vmatrix} -1 & 1 \\ -3 & 2 \end{vmatrix} = -1, & \Delta_{13} &= + \begin{vmatrix} -1 & 4 \\ -3 & 0 \end{vmatrix} = 12, \\ \Delta_{21} &= - \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = -6, & \Delta_{22} &= + \begin{vmatrix} 2 & 0 \\ -3 & 2 \end{vmatrix} = 4, & \Delta_{23} &= - \begin{vmatrix} 2 & 3 \\ -3 & 0 \end{vmatrix} = -9, \\ \Delta_{31} &= + \begin{vmatrix} 3 & 0 \\ 4 & 1 \end{vmatrix} = 3, & \Delta_{32} &= - \begin{vmatrix} 2 & 0 \\ -1 & 1 \end{vmatrix} = -2, & \Delta_{33} &= + \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} = 11. \end{aligned}$$

よって

$$(\Delta_{ij}) = \begin{pmatrix} 8 & -1 & 12 \\ -6 & 4 & -9 \\ 3 & -2 & 11 \end{pmatrix}$$

2 次の行列式を計算せよ. (各 2 点)

$$(1) \begin{vmatrix} 2 & 0 & 1 \\ -1 & -2 & 3 \\ 3 & 0 & 5 \end{vmatrix} \stackrel{\text{2で展開}}{=} 0\Delta_{12} - 2\Delta_{22} + 0\Delta_{32} = -2\Delta_{22} = -2 \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix} = -2(2 \times 5 - 1 \times 3) = -14.$$

$$(2) \begin{vmatrix} 0 & 1 & -3 & 0 \\ 0 & 3 & -2 & 3 \\ 0 & 2 & -1 & 0 \\ -3 & 2 & -1 & 2 \end{vmatrix} \stackrel{\text{1で展開}}{=} (-3)\Delta_{41} = (-3) \times \begin{vmatrix} 1 & -3 & 0 \\ 3 & -2 & 3 \\ 2 & -1 & 0 \end{vmatrix} = 3 \begin{vmatrix} 1 & -3 & 0 \\ 3 & -2 & 3 \\ 2 & -1 & 0 \end{vmatrix}$$

$$\stackrel{\text{3で展開}}{=} 3 \times 3 \begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix} = -9 \times (1 \times (-1) - (-3) \times 2) = -45.$$

$$(3) \begin{vmatrix} 0 & 1 & -2 & 1 \\ -1 & 0 & -1 & 0 \\ -2 & 1 & 0 & 1 \\ 1 & -1 & 2 & 0 \end{vmatrix} \stackrel{\text{1} \leftrightarrow \text{2}}{=} - \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 \\ -2 & 1 & 0 & 1 \\ 1 & -1 & 2 & 0 \end{vmatrix} \stackrel{\text{3} - 2 \times \text{1}}{\text{4} + \text{1}}{=} - \begin{vmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & 1 & 0 \end{vmatrix}$$

$$= -(-1) \begin{vmatrix} 1 & -2 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix} \stackrel{\text{2} - \text{1}}{\text{3} + \text{1}}{=} \begin{vmatrix} 1 & -2 & 1 \\ 0 & -4 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 0 \\ -1 & 1 \end{vmatrix} = 4 \times 1 = 4$$