

1 次の行列式を計算せよ. (各 1 点)

$$(1) \begin{vmatrix} 12 & 9 \\ 13 & 8 \end{vmatrix} \xrightarrow{\textcircled{2}-\textcircled{1}} \begin{vmatrix} 12 & 9 \\ 1 & -1 \end{vmatrix} = 3 \begin{vmatrix} 4 & 3 \\ 1 & -1 \end{vmatrix} = 3(4 \times (-1) - 3 \times 1) = -21$$

$$(2) \begin{vmatrix} 1 & -3 & 7 \\ 0 & 5 & 9 \\ 2 & -11 & 13 \end{vmatrix} \xrightarrow{\textcircled{3}-2 \times \textcircled{1}} \begin{vmatrix} 1 & -3 & 7 \\ 0 & 5 & 9 \\ 0 & -5 & -1 \end{vmatrix} \xrightarrow{\textcircled{3}+\textcircled{2}} \begin{vmatrix} 1 & -3 & 7 \\ 0 & 5 & 9 \\ 0 & 0 & 8 \end{vmatrix} = 1 \times 5 \times 8 = 40$$

$$(3) \begin{vmatrix} 9 & 3 & -2 \\ -6 & -8 & 1 \\ 3 & 0 & -1 \end{vmatrix} = 3 \begin{vmatrix} 3 & 3 & -2 \\ -2 & -8 & 1 \\ 1 & 0 & -1 \end{vmatrix} \xrightarrow{\textcircled{1} \leftrightarrow \textcircled{3}} -3 \begin{vmatrix} 1 & 0 & -1 \\ -2 & -8 & 1 \\ 3 & 3 & -2 \end{vmatrix} \xrightarrow{\begin{matrix} \textcircled{2}+2 \times \textcircled{1} \\ \textcircled{3}-3 \times \textcircled{1} \end{matrix}} -3 \begin{vmatrix} 1 & 0 & -1 \\ 0 & -8 & -1 \\ 0 & 3 & 1 \end{vmatrix} \\ = -3 \begin{vmatrix} -8 & -1 \\ 3 & 1 \end{vmatrix} = -3((-8) \times 1 - (-1) \times 3) = (-3) \times (-5) = 15$$

$$(4) \begin{vmatrix} 1 & 0 & -2 & -1 \\ 1 & 1 & -2 & 3 \\ 0 & 2 & -1 & 0 \\ -1 & 2 & -1 & 2 \end{vmatrix} \xrightarrow{\begin{matrix} \textcircled{2}-\textcircled{1} \\ \textcircled{4}+\textcircled{1} \end{matrix}} \begin{vmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 0 & 4 \\ 0 & 2 & -1 & 0 \\ 0 & 2 & -3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 4 \\ 2 & -1 & 0 \\ 2 & -3 & 1 \end{vmatrix} \xrightarrow{\begin{matrix} \textcircled{2}-2 \times \textcircled{1} \\ \textcircled{3}-2 \times \textcircled{1} \end{matrix}} \begin{vmatrix} 1 & 0 & 4 \\ 0 & -1 & -8 \\ 0 & -3 & -7 \end{vmatrix} \\ = \begin{vmatrix} -1 & -8 \\ -3 & -7 \end{vmatrix} = \begin{vmatrix} 1 & 8 \\ 3 & 7 \end{vmatrix} = 1 \times 7 - 8 \times 3 = 7 - 24 = -17$$

$$(5) \begin{vmatrix} 2 & 0 & 2 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 0 & 2 \end{vmatrix} \xrightarrow{\textcircled{2}+\textcircled{1}} 2 \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & -1 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 0 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 2 \end{vmatrix} \\ = 2(-1^2 \times (-1) - 1 \times 2^2) = 2 \times (-3) = -6$$

$$\begin{aligned}
 (6) \quad & \begin{vmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{vmatrix} \xrightarrow{\textcircled{1} \leftrightarrow \textcircled{3}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{vmatrix} \xrightarrow{\textcircled{2} \leftrightarrow \textcircled{3}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 \end{vmatrix} \\
 & \xrightarrow{\textcircled{3} \leftrightarrow \textcircled{4}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix} = -1 \times 2 \times 3 \times 4 = -24
 \end{aligned}$$

2 次の行列式を因数分解せよ. (各1点)

$$\begin{aligned}
 (1) \quad & \begin{vmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{vmatrix} \xrightarrow{\textcircled{4} - \textcircled{3}} \begin{vmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ 0 & 0 & 0 & d - c \end{vmatrix} \xrightarrow{\textcircled{3} - \textcircled{2}} \begin{vmatrix} a & a & a & a \\ a & b & b & b \\ 0 & 0 & c - b & c - b \\ 0 & 0 & 0 & d - c \end{vmatrix} \\
 & \xrightarrow{\textcircled{2} - \textcircled{1}} \begin{vmatrix} a & a & a & a \\ 0 & b - a & b - a & b - a \\ 0 & 0 & c - b & c - b \\ 0 & 0 & 0 & d - c \end{vmatrix} = a(b - a)(c - b)(d - c)
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \begin{vmatrix} a & a & b \\ a & b & a \\ b & a & a \end{vmatrix} \xrightarrow{\textcircled{1} + (\textcircled{2} + \textcircled{3})} \begin{vmatrix} 2a + b & 2a + b & 2a + b \\ a & b & a \\ b & a & a \end{vmatrix} = (2a + b) \begin{vmatrix} 1 & 1 & 1 \\ a & b & a \\ b & a & a \end{vmatrix} \\
 & \xrightarrow{\textcircled{2} - \textcircled{1}} \xrightarrow{\textcircled{3} - \textcircled{1}} (2a + b) \begin{vmatrix} 1 & 0 & 0 \\ a & b - a & 0 \\ b & a - b & a - b \end{vmatrix} = (2a + b)(b - a)(a - b) \\
 & = -(2a + b)(a - b)^2
 \end{aligned}$$

ポイント!

- 4次以上の行列式は(サラスの公式が使えないので)行列式の性質(線形性と交代性)を用いて計算する.
- 文字を含む行列式を因数分解するときは, 行列式の性質(線形性と交代性)を用いて計算すると良い.