

① 行列の基本変形を用いて, 連立1次方程式
$$\begin{cases} x + y + z = 12 \\ 3x - 2y - 2z = 6 \\ 4x + 3y - 5z = 2 \end{cases}$$
 を解け. (3点)

拡大係数行列 \tilde{A} は,

$$\begin{aligned} \tilde{A} &= \left(\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 3 & -2 & -2 & 6 \\ 4 & 3 & -5 & 2 \end{array} \right) \xrightarrow[\textcircled{3}-4\times\textcircled{1}]{\textcircled{2}-3\times\textcircled{1}}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & -5 & -5 & -30 \\ 0 & -1 & -9 & -46 \end{array} \right) \\ &\xrightarrow{\textcircled{2}\times(-\frac{1}{5})} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ 0 & 1 & 1 & 6 \\ 0 & -1 & -9 & -46 \end{array} \right) \xrightarrow[\textcircled{3}+\textcircled{2}]{\textcircled{1}-\textcircled{2}}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & -8 & -40 \end{array} \right) \\ &\xrightarrow{\textcircled{3}\times(-\frac{1}{8})} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & 5 \end{array} \right) \xrightarrow{\textcircled{2}-\textcircled{3}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{array} \right) \end{aligned}$$

よって $x = 6, y = 1, z = 5$.

② 次の行列を簡約化せよ. (各1点)

(1)
$$\begin{pmatrix} 1 & -3 \\ -6 & -5 \end{pmatrix} \xrightarrow{\textcircled{2}+6\times\textcircled{1}} \begin{pmatrix} 1 & -3 \\ 0 & -23 \end{pmatrix} \xrightarrow{\textcircled{2}\times(-1)/23} \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} \xrightarrow{\textcircled{1}+3\times\textcircled{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(2)
$$\begin{pmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 3 & -6 & 8 \end{pmatrix} \xrightarrow[\textcircled{3}-3\times\textcircled{1}]{\textcircled{2}+2\times\textcircled{1}}} \begin{pmatrix} 1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \xrightarrow{\textcircled{3}\times 1/2} \begin{pmatrix} 1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\textcircled{1}-2\times\textcircled{3}} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\textcircled{2}\leftrightarrow\textcircled{3}} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

(3)
$$\begin{pmatrix} 0 & 0 & 3 \\ 2 & 0 & 1 \\ -1 & 1 & -5 \end{pmatrix} \xrightarrow{\textcircled{1}\leftrightarrow\textcircled{3}} \begin{pmatrix} -1 & 1 & -5 \\ 2 & 0 & 1 \\ 0 & 0 & 3 \end{pmatrix} \xrightarrow{\textcircled{2}+2\times\textcircled{1}} \begin{pmatrix} -1 & 1 & -5 \\ 0 & 2 & -9 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\xrightarrow{\textcircled{3}\times 1/3} \begin{pmatrix} -1 & 1 & -5 \\ 0 & 2 & -9 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[\textcircled{2}+9\times\textcircled{3}]{\textcircled{1}+5\times\textcircled{3}}} \begin{pmatrix} -1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\textcircled{2}\times 1} \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\textcircled{1}-\textcircled{2}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\textcircled{1}\times(-1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3 次の連立1次方程式を基本変形(掃き出し法)を用いて解け. (各1点)

$$(1) \left(\begin{array}{cc|c} x & y & 3 \\ 1 & 1 & 3 \\ 3 & 2 & 5 \end{array} \right) \xrightarrow{\textcircled{2}-3\times\textcircled{1}} \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -1 & -4 \end{array} \right) \xrightarrow{\textcircled{2}\times(-1)} \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 4 \end{array} \right) \xrightarrow{\textcircled{1}-\textcircled{2}} \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 4 \end{array} \right)$$

よって $x = -1, y = 4$

$$(2) \left(\begin{array}{ccc|c} x & y & z & 1 \\ 4 & -1 & 8 & 1 \\ 2 & 0 & 5 & 1 \\ 0 & 1 & 2 & 1 \end{array} \right) \xrightarrow{\textcircled{1}\leftrightarrow\textcircled{2}} \left(\begin{array}{ccc|c} 2 & 0 & 5 & 1 \\ 4 & -1 & 8 & 1 \\ 0 & 1 & 2 & 1 \end{array} \right) \xrightarrow{\textcircled{2}-2\times\textcircled{1}} \left(\begin{array}{ccc|c} 2 & 0 & 5 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & 2 & 1 \end{array} \right)$$

$$\xrightarrow{\textcircled{3}+\textcircled{2}} \left(\begin{array}{ccc|c} 2 & 0 & 5 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\textcircled{1}\times 1/2, \textcircled{2}\times(-1)} \left(\begin{array}{ccc|c} 1 & 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$z = t$ とおけば, $\begin{cases} x = \frac{1-5t}{2} \\ y = 1-2t \\ z = t \end{cases} \quad (t \text{ は任意})$

$$(3) \left(\begin{array}{cccc|c} x & y & z & w & -7 \\ 1 & 1 & -2 & 0 & -7 \\ -2 & -2 & 5 & -2 & 17 \end{array} \right) \xrightarrow{\textcircled{2}+2\times\textcircled{1}} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & -7 \\ 0 & 0 & 1 & -2 & 3 \end{array} \right)$$

$$\xrightarrow{\textcircled{1}+2\times\textcircled{2}} \left(\begin{array}{cccc|c} 1 & 1 & 0 & -4 & -1 \\ 0 & 0 & 1 & -2 & 3 \end{array} \right)$$

y と w をパラメータにとり, それぞれ $y = t_1, w = t_2$ とおく. ②より $z - 2w = 3$. よって $z = 2t_2 + 3$. ①より

$$x = -y + 4w - 1 = -t_1 + 4t_2 - 1$$

故に

$$\begin{cases} x = -t_1 + 4t_2 - 1 \\ y = t_1 \\ z = 2t_2 + 3 \\ w = t_2 \end{cases} \quad (t_1, t_2 \text{ は任意})$$