

1 次の連立1次方程式を解け. (2点)

$$\left(\begin{array}{ccc|c} x & y & z & 2 \\ 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 2 \\ 1 & 3 & 5 & 4 \end{array} \right) \xrightarrow{\textcircled{3}-\textcircled{1}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 2 \end{array} \right) \xrightarrow{\begin{array}{l} \textcircled{1}-2\times\textcircled{2} \\ \textcircled{3}-\textcircled{2} \end{array}} \left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$z = t \text{ とおけば, } \begin{cases} x = -2 + t \\ y = 2 - 2t \\ z = t \end{cases} \quad (t \text{ は任意})$$

2 次の連立1次方程式の解の存在について答えよ. ただし, 答えは次の3つの中から選べ: (a) 解は存在しない; (b) 唯一の解が存在する; (c) 解は無数個存在する. (各2点)

$$\begin{aligned} & \left(\begin{array}{ccc|c} x & y & z & 1 \\ 0 & 2 & 1 & 1 \\ 3 & 2 & 3 & 2 \\ 2 & 1 & 2 & 2 \end{array} \right) \xrightarrow{\textcircled{1}\leftrightarrow\textcircled{2}} \left(\begin{array}{ccc|c} 3 & 2 & 3 & 2 \\ 0 & 2 & 1 & 1 \\ 2 & 1 & 2 & 2 \end{array} \right) \xrightarrow{\textcircled{1}-\textcircled{3}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 2 & 1 & 2 & 2 \end{array} \right) \\ (1) & \xrightarrow{\textcircled{3}-2\times\textcircled{1}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & -1 & 0 & 2 \end{array} \right) \xrightarrow{\textcircled{2}+2\times\textcircled{3}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & -1 & 0 & 2 \end{array} \right) \\ & \xrightarrow{\textcircled{2}\leftrightarrow\textcircled{3}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{array} \right) \end{aligned}$$

係数行列と拡大係数行列の階数が等しいので, 唯一の解が存在する.

答え: _____ (b)

$$(2) \quad \left(\begin{array}{ccc|c} x & y & z & 3 \\ 1 & 2 & 2 & 3 \\ 3 & 2 & 3 & 2 \\ 6 & 8 & 9 & 1 \end{array} \right) \xrightarrow{\begin{array}{l} \textcircled{2}-3\times\textcircled{1} \\ \textcircled{3}-6\times\textcircled{1} \end{array}} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & -4 & -3 & -7 \\ 0 & -4 & -3 & -17 \end{array} \right) \xrightarrow{\textcircled{3}-\textcircled{2}} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & -4 & -3 & -7 \\ 0 & 0 & 0 & -10 \end{array} \right)$$

拡大係数行列の階数が係数行列の階数より大きいので, 方程式の解は存在しない.

答え: _____ (a)