対称式の基本対称式による表示

代数学序論（東海大学情報数理学科, 担当：那須）

対称式を基本対称式で表す命令

SymmetricReduction[f, {x_1, …, x_n}, {s_1, …, s_n}]
変数x_1, …, x_nに関する対称式p を基本対称式 s_1, …, s_nで置き換えたペア{p, q}を与える

2変数(Binomial symmmetric functions)

2次式

```
SymmetricReduction[x^2 + y^2, {x, y}, {s1, s2}]
```

$\{s1^2 - 2\,s2,\ 0\}$

3次式

```
SymmetricReduction[x^3 + y^3, {x, y}, {s1, s2}]
```

$\{s1^3 - 3\,s1\,s2,\ 0\}$

4次式

```
SymmetricReduction[x^4 + y^4, {x, y}, {s1, s2}]
```

$\{s1^4 - 4\,s1^2\,s2 + 2\,s2^2,\ 0\}$

5次式

```
SymmetricReduction[x^5 + y^5, {x, y}, {s1, s2}]
```

$\{s1^5 - 5\,s1^3\,s2 + 5\,s1\,s2^2,\ 0\}$

3変数

2次式

```
SymmetricReduction[x^2 + y^2 + z^2, {x, y, z}, {s1, s2, s3}]
```

$\{s1^2 - 2\,s2,\ 0\}$

```
SymmetricReduction[(x - y)^2 + (y - z)^2 + (z - x)^2, {x, y, z}, {s1, s2, s3}]
```

$\{2\,s1^2 - 6\,s2,\ 0\}$

```
SymmetricReduction[(x + y)^2 + (y + z)^2 + (z + x)^2, {x, y, z}, {s1, s2, s3}]
```

$\{2\,s1^2 - 2\,s2,\ 0\}$

```
SymmetricReduction[Simplify[Det[{{x, y, z}, {x^3, y^3, z^3}, {x^4, y^4, z^4}}] /
   Det[{{x, y, z}, {x^2, y^2, z^2}, {x^3, y^3, z^3}}]], {x, y, z}, {s1, s2, s3}]
```

$$\{s2,\ 0\}$$

```
SymmetricReduction[Simplify[Det[{{x, y, z}, {x^2, y^2, z^2}, {x^5, y^5, z^5}}] /
   Det[{{x, y, z}, {x^2, y^2, z^2}, {x^3, y^3, z^3}}]], {x, y, z}, {s1, s2, s3}]
```

$$\{s1^2 - s2,\ 0\}$$

3次式

```
SymmetricReduction[x^3 + y^3 + z^3, {x, y, z}, {s1, s2, s3}]
```

$$\{s1^3 - 3\ s1\ s2 + 3\ s3,\ 0\}$$

```
SymmetricReduction[z (x - y)^2 + x (y - z)^2 + y (z - x)^2, {x, y, z}, {s1, s2, s3}]
```

$$\{s1\ s2 - 9\ s3,\ 0\}$$

```
SymmetricReduction[z (x + y)^2 + x (y + z)^2 + y (z + x)^2, {x, y, z}, {s1, s2, s3}]
```

$$\{s1\ s2 + 3\ s3,\ 0\}$$

```
SymmetricReduction[
 z (x^2 + y^2) + x (y^2 + z^2) + y (z^2 + x^2), {x, y, z}, {s1, s2, s3}]
```

$$\{s1\ s2 - 3\ s3,\ 0\}$$

```
SymmetricReduction[z^2 (x + y) + x^2 (y + z) + y^2 (z + x), {x, y, z}, {s1, s2, s3}]
```

$$\{s1\ s2 - 3\ s3,\ 0\}$$

```
SymmetricReduction[(x + y)^3 + (y + z)^3 + (z + x)^3, {x, y, z}, {s1, s2, s3}]
```

$$\{2\ s1^3 - 3\ s1\ s2 - 3\ s3,\ 0\}$$

```
SymmetricReduction[(x + y) (y + z) (z + x) + x * y * z, {x, y, z}, {s1, s2, s3}]
```

$$\{s1\ s2,\ 0\}$$

```
SymmetricReduction[Det[{{x, y, z}, {z, x, y}, {y, z, x}}], {x, y, z}, {s1, s2, s3}]
```

$$\{s1^3 - 3\ s1\ s2,\ 0\}$$

```
SymmetricReduction[
 Simplify[Det[{{x^2, y^2, z^2}, {x^3, y^3, z^3}, {x^4, y^4, z^4}}] /
   Det[{{x, y, z}, {x^2, y^2, z^2}, {x^3, y^3, z^3}}]], {x, y, z}, {s1, s2, s3}]
```

$$\{s3,\ 0\}$$

4次式

```
SymmetricReduction[x^4 + y^4 + z^4, {x, y, z}, {s1, s2, s3}]
```

$$\{s1^4 - 4\ s1^2\ s2 + 2\ s2^2 + 4\ s1\ s3,\ 0\}$$

```
SymmetricReduction[x^2 y^2 + y^2 z^2 + z^2 x^2, {x, y, z}, {s1, s2, s3}]
```

$$\{s2^2 - 2\ s1\ s3,\ 0\}$$

```
SymmetricReduction[(x - y) ^ 4 + (y - z) ^ 4 + (z - x) ^ 4, {x, y, z}, {s1, s2, s3}]
```

$$\{2\,s1^4 - 12\,s1^2\,s2 + 18\,s2^2,\ 0\}$$

```
SymmetricReduction[(x + y) ^ 4 + (y + z) ^ 4 + (z + x) ^ 4, {x, y, z}, {s1, s2, s3}]
```

$$\{2\,s1^4 - 4\,s1^2\,s2 + 2\,s2^2 - 8\,s1\,s3,\ 0\}$$

```
SymmetricReduction[
 x ^ 2 (y - z) ^ 2 + y ^ 2 * (z - x) ^ 2 + z ^ 2 * (x - y) ^ 2, {x, y, z}, {s1, s2, s3}]
```

$$\{2\,s2^2 - 6\,s1\,s3,\ 0\}$$

```
SymmetricReduction[
 x ^ 2 (y + z) ^ 2 + y ^ 2 * (z + x) ^ 2 + z ^ 2 * (x + y) ^ 2, {x, y, z}, {s1, s2, s3}]
```

$$\{2\,s2^2 - 2\,s1\,s3,\ 0\}$$

```
SymmetricReduction[x (y + z) ^ 3 + y * (z + x) ^ 3 + z * (x + y) ^ 3, {x, y, z}, {s1, s2, s3}]
```

$$\{s1^2\,s2 - 2\,s2^2 + 5\,s1\,s3,\ 0\}$$

```
SymmetricReduction[x ^ 3 (y + z) + y ^ 3 * (z + x) + z ^ 3 * (x + y), {x, y, z}, {s1, s2, s3}]
```

$$\{s1^2\,s2 - 2\,s2^2 - s1\,s3,\ 0\}$$

```
SymmetricReduction[
 x * y * (x - y) ^ 2 + y * z * (y - z) ^ 2 + z * x * (z - x) ^ 2, {x, y, z}, {s1, s2, s3}]
```

$$\{s1^2\,s2 - 4\,s2^2 + 3\,s1\,s3,\ 0\}$$

```
SymmetricReduction[Simplify[Det[{{x, y, z}, {x^4, y^4, z^4}, {x^5, y^5, z^5}}] /
    Det[{{x, y, z}, {x^2, y^2, z^2}, {x^3, y^3, z^3}}]], {x, y, z}, {s1, s2, s3}]
```

$$\{s2^2 - s1\,s3,\ 0\}$$

5次式

```
SymmetricReduction[x ^ 5 + y ^ 5 + z ^ 5, {x, y, z}, {s1, s2, s3}]
```

$$\{s1^5 - 5\,s1^3\,s2 + 5\,s1\,s2^2 + 5\,s1^2\,s3 - 5\,s2\,s3,\ 0\}$$

4変数

2次式

```
SymmetricReduction[x ^ 2 + y ^ 2 + z ^ 2 + w ^ 2, {x, y, z, w}, {s1, s2, s3, s4}]
```

$$\{s1^2 - 2\,s2,\ 0\}$$

```
SymmetricReduction[(x - y) ^ 2 + (y - z) ^ 2 + (z - x) ^ 2 + (x - w) ^ 2 + (y - w) ^ 2 + (z - w) ^ 2,
 {x, y, z, w}, {s1, s2, s3, s4}]
```

$$\{3\,s1^2 - 8\,s2,\ 0\}$$

```
SymmetricReduction[(x + y) ^ 2 + (y + z) ^ 2 + (z + x) ^ 2 + (x + w) ^ 2 + (y + w) ^ 2 + (z + w) ^ 2,
 {x, y, z, w}, {s1, s2, s3, s4}]
```

$$\{3\,s1^2 - 4\,s2,\ 0\}$$

3次式

```
SymmetricReduction[x^3 + y^3 + z^3 + w^3, {x, y, z, w}, {s1, s2, s3, s4}]
```

$$\{s1^3 - 3\, s1\, s2 + 3\, s3,\ 0\}$$

```
SymmetricReduction[(x + y - z - w) (x - y + z - w) (x - y - z + w),
 {x, y, z, w}, {s1, s2, s3, s4}]
```

$$\{s1^3 - 4\, s1\, s2 + 8\, s3,\ 0\}$$