

1 (1) 行列 $A = \begin{pmatrix} 2 & -3 & -2 \\ 3 & -2 & -2 \\ -2 & 4 & 1 \end{pmatrix}$ の (i, j) 余因子 Δ_{ij} ($1 \leq i, j \leq 3$) を全て求めよ. (9 点)

$$\begin{aligned} \Delta_{11} &= + \begin{vmatrix} -2 & -2 \\ 4 & 1 \end{vmatrix} = 6 & \Delta_{12} &= - \begin{vmatrix} 3 & -2 \\ -2 & 1 \end{vmatrix} = 1 & \Delta_{13} &= + \begin{vmatrix} 3 & -2 \\ -2 & 4 \end{vmatrix} = 8 \\ \Delta_{21} &= - \begin{vmatrix} -3 & -2 \\ 4 & 1 \end{vmatrix} = -5 & \Delta_{22} &= + \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = -2 & \Delta_{23} &= - \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = -2 \\ \Delta_{31} &= + \begin{vmatrix} -3 & -2 \\ -2 & -2 \end{vmatrix} = 2 & \Delta_{32} &= - \begin{vmatrix} 2 & -2 \\ 3 & -2 \end{vmatrix} = -2 & \Delta_{33} &= + \begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix} = 5 \end{aligned}$$

よって, $(\Delta_{ij}) = \begin{pmatrix} 6 & 1 & 8 \\ -5 & -2 & -2 \\ 2 & -2 & 5 \end{pmatrix}$

(2) A の行列式 $|A|$ の値を求めよ. (1 点)

$$\begin{aligned} |A| &= \frac{\textcircled{1} + 2 \times \textcircled{3}}{\textcircled{2} + 2 \times \textcircled{3}} \begin{vmatrix} -2 & 5 & 0 \\ -1 & 6 & 0 \\ -2 & 4 & 1 \end{vmatrix} \stackrel{\textcircled{3} \text{で展開}}{=} 1 \times \Delta_{33} = \begin{vmatrix} -2 & 5 \\ -1 & 6 \end{vmatrix} \\ &= (-2) \times 6 - 5 \times (-1) = -7 \end{aligned}$$

(3) A の逆行列 A^{-1} を求めよ. (1 点)

$$A^{-1} = \frac{1}{|A|} {}^t(\Delta_{ij}) = \frac{1}{-7} \begin{pmatrix} 6 & -5 & 2 \\ 1 & -2 & -2 \\ 8 & -2 & 5 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} -6 & 5 & -2 \\ -1 & 2 & 2 \\ -8 & 2 & -5 \end{pmatrix}$$