

1 次の行列式を計算せよ. (ただし (3) の答えは, 1 次式の積に因数分解した形で答えよ.) (各 1 点)

$$\begin{aligned}
 (1) \quad & \begin{vmatrix} 9 & 3 & -2 \\ -6 & -8 & 1 \\ 3 & 0 & -1 \end{vmatrix} = 3 \begin{vmatrix} 3 & 3 & -2 \\ -2 & -8 & 1 \\ 1 & 0 & -1 \end{vmatrix} \xrightarrow{\textcircled{1} \leftrightarrow \textcircled{3}} -3 \begin{vmatrix} 1 & 0 & -1 \\ -2 & -8 & 1 \\ 3 & 3 & -2 \end{vmatrix} \xrightarrow{\begin{matrix} \textcircled{2} + 2 \times \textcircled{1} \\ \textcircled{3} - 3 \times \textcircled{1} \end{matrix}} -3 \begin{vmatrix} 1 & 0 & -1 \\ 0 & -8 & -1 \\ 0 & 3 & 1 \end{vmatrix} \\
 & = -3 \begin{vmatrix} -8 & -1 \\ 3 & 1 \end{vmatrix} = -3((-8) \times 1 - (-1) \times 3) = (-3) \times (-5) = 15
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \begin{vmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{vmatrix} \xrightarrow{\textcircled{1} \leftrightarrow \textcircled{3}} - \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{vmatrix} \xrightarrow{\textcircled{2} \leftrightarrow \textcircled{3}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 \end{vmatrix} \\
 & \xrightarrow{\textcircled{3} \leftrightarrow \textcircled{4}} - \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix} = -1 \times 2 \times 3 \times 4 = -24
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & \begin{vmatrix} a & a & b \\ a & b & a \\ b & a & a \end{vmatrix} \xrightarrow{\textcircled{1} + (\textcircled{2} + \textcircled{3})} \begin{vmatrix} 2a + b & 2a + b & 2a + b \\ a & b & a \\ b & a & a \end{vmatrix} = (2a + b) \begin{vmatrix} 1 & 1 & 1 \\ a & b & a \\ b & a & a \end{vmatrix} \\
 & \xrightarrow{\begin{matrix} \textcircled{2} - \textcircled{1} \\ \textcircled{3} - \textcircled{1} \end{matrix}} (2a + b) \begin{vmatrix} 1 & 0 & 0 \\ a & b - a & 0 \\ b & a - b & a - b \end{vmatrix} = (2a + b)(b - a)(a - b) \\
 & = -(2a + b)(a - b)^2
 \end{aligned}$$

ポイント!

文字を含む行列式を因数分解するときは, 行列式の性質 (線形性と交代性) を用いて計算すると良い.