

1 掃き出し法を用いて, 次の行列 A の逆行列 A^{-1} を求めよ. ただし A^{-1} が存在しない場合には, 「存在しない」と答えよ. (各 1 点)

$$(1) A = \begin{pmatrix} 1 & 2 \\ 4 & 7 \end{pmatrix} \xrightarrow{\textcircled{2}-4\times\textcircled{1}} \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} \xrightarrow{\textcircled{1}+2\times\textcircled{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \xrightarrow{(-1)\times\textcircled{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{\textcircled{2}-4\times\textcircled{1}} \begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix} \xrightarrow{\textcircled{1}+2\times\textcircled{2}} \begin{pmatrix} -7 & 2 \\ -4 & 1 \end{pmatrix} \xrightarrow{(-1)\times\textcircled{2}} \begin{pmatrix} -7 & 2 \\ 4 & -1 \end{pmatrix}$$

よって, $A^{-1} = \begin{pmatrix} -7 & 2 \\ 4 & -1 \end{pmatrix}$

$$(2) A = \begin{pmatrix} 4 & -14 \\ -2 & 7 \end{pmatrix} \xrightarrow{\textcircled{1}\leftrightarrow\textcircled{2}} \begin{pmatrix} -2 & 7 \\ 4 & -14 \end{pmatrix} \xrightarrow{\textcircled{2}+2\times\textcircled{1}} \begin{pmatrix} -2 & 7 \\ 0 & 0 \end{pmatrix}$$

rank $A = 1 < 2$ より, A^{-1} は存在しない.

$$(3) A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & -1 & 1 \\ 1 & 2 & -1 \end{pmatrix} \xrightarrow{\begin{matrix} \textcircled{2}+\textcircled{1} \\ \textcircled{3}-\textcircled{1} \end{matrix}} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{\textcircled{2}\leftrightarrow\textcircled{3}} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \xrightarrow{\begin{matrix} \textcircled{1}-2\times\textcircled{3} \\ \textcircled{2}+\textcircled{3} \end{matrix}} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\xrightarrow{\textcircled{1}-\textcircled{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \xrightarrow{(-1)\times\textcircled{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\begin{matrix} \textcircled{2}+\textcircled{1} \\ \textcircled{3}-\textcircled{1} \end{matrix}} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \xrightarrow{\textcircled{2}\leftrightarrow\textcircled{3}} \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \xrightarrow{\begin{matrix} \textcircled{1}-2\times\textcircled{3} \\ \textcircled{2}+\textcircled{3} \end{matrix}} \begin{pmatrix} -1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\textcircled{1}-\textcircled{2}} \begin{pmatrix} -1 & -3 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \xrightarrow{(-1)\times\textcircled{3}} \begin{pmatrix} -1 & -3 & -1 \\ 0 & 1 & 1 \\ -1 & -1 & 0 \end{pmatrix}$$

よって $A^{-1} = \begin{pmatrix} -1 & -3 & -1 \\ 0 & 1 & 1 \\ -1 & -1 & 0 \end{pmatrix}$

ポイント!

正方行列 A に対し,

$$(A|E) \longrightarrow (E|B)$$

と基本変形されるならば, B は A の逆行列 A^{-1} に等しい.